On one problem of American option pricing

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Consider (B_n, S_n)-market, whose dynamic describe by the model

$$\begin{aligned} B_n &= (1+r)B_{n-1}, \quad n \geq 1, \quad B_0 > 0; \\ S_n &= (1+\rho_n)S_{n-1}, \quad n \geq 1, \quad S_0 > 0, \end{aligned} \tag{1}$$

where $r > 0$ is compound interest rate, ρ_n take values $\lambda^{-1} - 1$ or $\lambda - 1, \lambda > 0$. The payoff of an option is given by the expression [1]: $f_{\tau(\omega)}(\omega) = \beta^{\tau(\omega)} \max_{k \leq \tau} S_k(\omega)$. In investent process we are consider the following type of income (expenses) $g_n = c_1 \beta_n B_{n-1} + c_2 \gamma_n S_{n-1}$, where $\pi_n = (\beta_n, \gamma_n)$ is non-self-finansing portfolio, $0 < c_1 < 1, 0 < c_2 < 1$.

The fair price of an option can be written in the form:

$$C^* = \sup_\tau E^* \alpha^\tau f_\tau = S_0 \sup_\tau \widetilde{E} \beta^\tau X_\tau, \tag{2}$$

where the expectations are taken by the directly constructed martingale measures $\widetilde{P}$ and P^*, and X_n is some markov sequence.

Theorem. The following statements are fulfilled:

a) optimal stopping moment have the following form

$$\tau^* = inf\{n \geq 0 : X_n \in [\lambda^k, \infty)\}, \quad k > 0, \tag{3}$$

b) the fair price can be computed by the equality

$$C^* = S_0 c, \quad c > 0. \tag{4}$$

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References

[1]. Д. О. Крамков., А. Н. Ширяев., О расчетах рациональной стоимости “Русского опциона” в симметричной биномиальной модели (B,S)-рынка, ТВП, 39:1 (1994), 191–200.