On almost everywhere divergence of generalized Cesáro means of trigonometric Fourier series

Teimuraz Akhobadze, Shalva Zviadadze

e-mail: takhoba@gmail.com
e-mail: sh.zviadadze@gmail.com
Department of Mathematics, Faculty of Exact and Natural Sciences, Ivane Javakhishvili Tbilisi State University, 13, University St., Tbilisi, Georgia

Let (α_n) and (S_n) be a sequence of real numbers, where $\alpha_n > -1$, $n \in \mathbb{N}$, and

$$\sigma_n^{\alpha_n} \equiv \sum_{\nu=0}^{n} A_{n-\nu}^{\alpha_n - 1} S_\nu / A_n^{\alpha_n}, \tag{1}$$

where

$$A_k^{\alpha_n} = (\alpha_n + 1)(\alpha_n + 2) \cdot \ldots \cdot (\alpha_n + k)/k!.$$

It is clear that $\sigma_n^0 = S_n$. If (α_n) is a constant sequence $(\alpha_n = \alpha,\ n \in \mathbb{N})$ then $\sigma_n^{\alpha_n}$ coincides with the usual Cesáro σ_n^{α}-means [10]. If in (1) instead of S_ν we substitute partial sums $S_\nu(f, x)$ of the Fourier series of a function f with respect to the trigonometric system then the corresponding means $\sigma_n^{\alpha_n}$ is denoted by $\sigma_n^{\alpha_n}(f, x)$.

These means were studied by Kaplan [4]. The author compared the methods of summability (C, α_n) and (C, α), and obtained necessary and sufficient conditions, in terms of the α_n, for the inclusion $(C, \alpha_n) \subset (C, \alpha)$, and sufficient conditions for $(C, \alpha) \subset (C, \alpha_n)$. Later Akhobadze ([1]-[3]) and Tetunashvili ([6]-[9]) investigated problems of (C, α_n) summability of trigonometric Fourier series.

In our talk we explore behaviour of (C, α_n)-means of trigonometric Fourier series of integrable functions for sequences α_n. In particular we extend Kolmogorov's [5] well known theorem on a.e. divergence of trigonometric Fourier series.

Theorem 1. *For any $\alpha_n \to 0+$, $n \to +\infty$, there exists function $f \in L[0; 2\pi]$ such that*

$$\limsup_{n \to +\infty} |\sigma_n^{\alpha_n}(f, x)| = +\infty \quad \textit{almost everwhere.}$$

Theorem 2. *Let $f \in L(0; 2\pi)$ and $\alpha_n \to 0$, $n \to +\infty$. Then for almost every $x \in (0; 2\pi)$*

$$\lim_{n \to +\infty} \alpha_n \sigma_n^{\alpha_n}(f, x) = 0.$$

Remark 3. *Let $\alpha_n \downarrow 0$ and $\alpha_n \cdot \ln n \uparrow +\infty$, $n \to +\infty$. Then for any sequence ψ_n such that $\psi_n \uparrow +\infty$ and $\alpha_n \cdot \psi_n \to 0$, $n \to +\infty$, there exists a function f for which*

$$\limsup_{n \to +\infty} |\alpha_n \cdot \psi_n \cdot \sigma_n^{\alpha_n}(f, x)| = +\infty \quad \textit{almost everywhere.}$$

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