SOME CLASSES OF SIMPLE SETS

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Tennenbaum (see, [3, p.159]) defined the notion of Q-reducibility on stes of natural numbers and Friedberg and Rogers [1] defined the notion of s-reducibility.

If $A \leq_s B$ via a computable function f such that for all $x, y, x \neq y \Rightarrow W_{f(x)} \cap W_{f(y)} = \emptyset$ and $\bigcup_{x\epsilon\omega} W_{f(x)}$ is computable, then we say that A is $s_{1,N}$ *-reducible* to B (in symbols: $A \leq_{s_{1,N}} B$).

If, in addition, $(\forall x)(W_{f(x)}$ is finite), then we say that A is $s_{1,N,f}$ $-$ *reducible* to B (in symbols: $A \leq_{s_{1,N,f}} B$).

Our notation and terminology are standard and can be found in [2] and [3].

Theorem 1. Let A be a coinfinite c.e. set such that A does not belong to the class **sHS**. Then there is a c.e. set C, $C \subseteq A$, such that every c.e. coinfinite superset of the set C is Q-complete.

Corollary. SH $\subseteq$ sHS .

Theorem 2. For every noncomputable c.e. set C there exist c.e. sets A, S such that A is a hypersimple, S is a simple nonhypersimple and

$$C \equiv_T A \;\&\; A \equiv_Q S \;\&\; S \not\leq_{tt} A.$$

Theorem 3. Let K be a creative set and let A be an arbitrary infinite set. Then A is strongly hyperimmune (finitely strongly hyperimmune) if and on ly if $\overline{K} \not\leq s_{1,N}\, B$ ($\overline{K} \not\leq s_{1,N,f}\, B$) for all infinite subset B of A.

Theorem 4. Let A be a Σ^0_2 infinite set and let K be a creative set. Then A is strongly hyperimmune if and only if $\overline{K} \not\leq s_{1,N}\, B$ for all Σ^0_2 (equivalently, Δ^0_2) infinite subset B of A.

Theorem 5. Let A be a Σ^0_2 infinite set and let K be a creative set. Then A is finitely strongly hyperimmune if and only if $\overline{K} \not\leq s_{1,N,f}$ B for all Σ^0_2 (equivalently, Δ^0_2) infinite subset B of A.

[1]] R.M.Friedberg, H.Rogers, Jr., Reducibility and completeness for sets of integers, Z.Math. Logik Grundlag. Math.1959, 5.

[2] E.Herrmann, Classes of simple sets, filter properties and their mutual postion, Seminarber.Humbolt-Univ. Berlin. Sekt. Math. 1984, 60.

[3] H.Rogers, Theory of recursive functions and effective computability. McGraw-Hill Book Co., New York, 1967.